4.8 Using Matrices to Solve Systems of Equations

Matrix Equation

$$\begin{bmatrix}1 & 3\\ 1 & 2\end{bmatrix}\cdot\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}3\\ 7\end{bmatrix}$$

$$\begin{bmatrix}x+3y\\ x+2y\end{bmatrix}=\begin{bmatrix}3\\ 7\end{bmatrix}\quad\begin{cases}x+3y=3\\ x+2y=7\end{cases}$$

Matrix Equation

$$\begin{bmatrix}1 & 3\\ 1 & 2\end{bmatrix}\cdot\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}3\\ 7\end{bmatrix}$$

Coefficient Matrix | Variable Matrix | Constant Matrix

Put the following system into a matrix equation:

5x - 3y = 9
2x + 3y = 7

$$\begin{bmatrix}5 & -3\\ 2 & 3\end{bmatrix}\cdot\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}9\\ 7\end{bmatrix}$$

Use Matrix equation to solve

A-coefficient matrix
X- variable matrix
B- constant matrix

a x = b

$\frac{1}{a}ax=\frac{1}{a}b$

$1x=\frac{1}{a}b$

$x=\frac{1}{a}b$

A X = B

$A^{-1}AX=A^{-1}B$

$IX=A^{-1}B$

$X=A^{-1}B$

ex:
5x + 3y = 13
4x + 7y = -8

$A^{-1} \cdot B$

(5, -4)

$\begin{bmatrix} 5 & 3 \\ 4 & 7 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 13 \\ -8 \end{bmatrix}$

A B

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -4 \end{bmatrix}$

ex:
5x - 2y = 3
8x + 4y = 3

$\begin{bmatrix} 5 & -2 \\ 8 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$

$\left(\frac{1}{2}, -\frac{1}{4}\right)$

ex:
10x + 5y = 15
6x + 3y = -6

$2x + y = 3$
$-6x - 3y = -9$

Either $\emptyset$ or ∞

$\emptyset$

ex:
3x - 2y + z = 0
2x +3y - z = 17
5x -y + 4z = -7

(3, 2, -5)

$\begin{bmatrix} 3 & -2 & 1 \\ 2 & 3 & -1 \\ 5 & -1 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 17 \\ -7 \end{bmatrix}$

HW
p205-206
4-6, 24, 26-28